

LOST POTENTIAL TRACKER

# METHODOLOGY NOTE

## APRIL 2021



# ACKNOWLEDGEMENTS

The ONE Campaign would like to thank the following individuals for contributing to the report.

Alasdair Mackintosh and Paul Atherton from Fab Inc. were hired as consultants and spearheaded the methodology, conducted all data analysis, and wrote the report. This analysis would have been impossible without them.

We would like to thank the members of the Advisory Council for their guidance in shaping the methodology: Manos Antoninis (UNESCO), Joao Pedro Azevedo (World Bank), Jessica Bergmann (Education Commission), Matt Brossard (UNICEF), Elise Wendlassida Miningou (Global Partnership for Education), Marla Spivack, Morgan Hanadi Strecker (Global Partnership for Education), and Hollie Warren (Save the Children UK). A special thank you to Joao Pedro de Azevedo and Elise Wendlassida Miningou for the many hours they contributed to the impact of financing on the figures in this report.

At ONE, the report's direction and editing was led by Natasha Somji. David McNair and Jorge Rivera were also involved in providing feedback on various areas of the report. The report was designed by Kavin Chhoeut.

# TABLE OF CONTENTS

Table of Figures .....	4
List of Acronyms .....	5
<b>1. Introduction .....</b>	<b>6</b>
1.1. Overview .....	6
1.2. Purpose .....	6
1.3. Limitations .....	6
1.4. Contents .....	6
<b>2. Methodology for Calculating Lost Potential and Secured Potential .....</b>	<b>6</b>
2.1. Data Overview .....	7
2.2. Learning Poverty Data .....	7
2.3. Lost Potential .....	7
2.4. Secured Potential .....	8
<b>3. Scenarios Methodology .....</b>	<b>8</b>
3.1. Pre-COVID-19 Modeling .....	8
3.2. Post-COVID-19 Modeling .....	8
<b>4. Methodology for Returns to Financing on Lost Potential .....</b>	<b>9</b>
4.1. Financing Overview .....	9
4.2. Data and Descriptive Statistics .....	10
4.3. Stochastic Frontier Model results .....	13
4.4. Outcomes and Bootstrap Model Results .....	14
4.5. Efficiency of Spending .....	15
4.6. Targeting of Spending and the Learning Poverty Gap .....	16
4.7. Financing Conclusion .....	17
<b>5. Bounds on changing financing .....</b>	<b>17</b>
5.1. Additional financing .....	17
<b>6. External benefits to other SDGs .....</b>	<b>18</b>
6.1. Cost of one additional year of LAYS .....	19
6.2. GDP (SDG 8) .....	19
6.3. Extreme Poverty (SDG 1) .....	21
6.4. Government Revenue (SDG 17) .....	21
6.5. Health (SDG 3) .....	21
6.6. Early Marriage and Gender (SDG 5) .....	22
6.7. Nutrition (SDG 2) .....	22
Appendix A .....	23
Appendix B .....	24
Endnotes .....	27

# TABLE OF FIGURES

Table 1: Learning Poverty Data .....	7
Table 2: Impact of COVID-19 on learning poverty rates by income group (percentage points).....	8
Table 3: Descriptive statistics .....	12
Table 4: Variation in key variables.....	12
Table 5: Results of the stochastic frontier estimations.....	13
Table 6: Calculating outcomes from the regression outputs .....	14
Table 7: Bootstrapped estimates for 1,000 repetitions .....	15
Table 8: The effect of financing on lost potential at full efficiency of spending .....	16
Table 9: The effect of financing on lost potential at full efficiency of spending and targeting the learning poor at their distance from being able to read by age 10 .....	17
Table 10: Estimating bounds for additions in annual financing.....	18
Table 11: Cost of one additional year of LAYS (USD) .....	19
Table 12: Change in LAYS based on example financing.....	19
Table 13: External benefits to GDP (USD) .....	20
Table 14: External benefits to extreme poverty reduction (USD).....	21
Table 15: External benefits to public revenue generation (USD).....	21
Table 16: External benefits to health (USD).....	22
Table 17: External benefits to reducing early marriage .....	22
Table 18: External benefits to reducing child stunting .....	23
Table 19: Results of the stochastic frontier estimations with multiple observations allowed per country, weighted by the number of observations per country. ....	24
Table 20: Weighted bootstrap estimates.....	24
Table 21: Results of the stochastic frontier estimations without the squared spend term, latest data only .....	25
Table 22: Bootstrap estimates without squared spend term .....	26

# LIST OF ACRONYMS

BAU	Business As Usual
DAC	Development Assistance Committee
DLP	Daily Lost Potential
DSP	Daily Secured Potential
EAP	East Asia and Pacific
ECA	East and Central Asia
EECA	Eastern Europe and Central Asia
ESA	Eastern and Southern Africa
GPE	Global Partnership for Education
HIC	High Income Country
LAC	Latin America and Caribbean
LAYS	Learning Adjusted Years of Schooling
LIC	Low Income Country
LMIC	Lower Middle Income Country
LP	Lost Potential
LPT	Lost Potential Tracker
MENA	Middle East and North Africa
NA	North America
ODA	Overseas Development Assistance
PPP	Purchasing Power Parity
SA	South Asia
SSA	Sub Saharan Africa
UIS	UNESCO Institute of Statistics
UMIC	Upper Middle Income Country
USD	United States Dollars
WCA	West and Central Africa
WE	Western Europe

## 1. Introduction

### 1.1 Overview

Reading proficiently by age 10 is a make-or-break benchmark in a child's development. Up until age 10, children are learning to read; but after this critical age, they should be switching to reading to learn.<sup>1</sup> If they are unable to read and comprehend at this age, it impedes their learning trajectory for the rest of their lives, spilling over into health, poverty alleviation and equality. This is referred to as learning poverty.

The Lost Potential Tracker (LPT) is an advocacy tool that aggregates how many children every second, minute, hour, day, week, month, and year have lost their potential through learning poverty. A child's potential is considered lost if they cannot read or comprehend a simple story for every unit of time past age 10. Every child that reaches their 10th birthday has crossed a boundary after which, in their subsequent days of life, they are not reaching their full potential - they are experiencing lost potential. The LPT also shows how many children have their potential secured, which is the inverse of lost potential or how many children reach their 10<sup>th</sup> birthday and can read and understand a simple story.

The LPT estimates the number of children experiencing lost potential and secured potential under a business as usual scenario. The potential impact of COVID-19 on lost potential is also investigated by using the 'pessimistic scenario' within World Bank estimates. Moreover, we also seek to estimate how additional education financing (whether from governments or donors) can reduce the numbers of children with lost potential, and how this additional financing on education can also bring about wider benefits across other sectors in the long-term.

### 1.2 Purpose

The key purpose of the LPT is to drive a sense of urgency around the global learning crisis. This methodology aims to provide a robust basis for the estimation of how many children are experiencing lost potential and secured potential over time and identify how this can change based on education financing and the wider impacts that this would have on other Sustainable Development Goals (SDGs).

### 1.3 Limitations

The greatest limitation to this methodology is that this is still a nascent area for research, and as such there are limited peer reviewed methodologies to draw on. In addition, what does exist is often based on cross-country growth regressions, which have limitations in terms of their ability to speak causally to changes arising from education. The learning poverty indicator, on which the LPT is based, was launched in late 2019 and the Learning Adjusted Years of Schooling (LAYS) measure, which also underpins the modeling for the wider long-term benefits, was developed in 2018. The body of literature involving these is therefore still developing. More specific limitations of the work are raised in subsequent sections.

### 1.4 Contents

This document is split into four main sections. First, the methodology for the LPT and related key flow measures are introduced. Second, we bring in estimates of the impact of COVID-19 on the LPT. Third, the methodology for estimating how increases in education financing would affect the LPT are covered. Finally, we broaden the outlook to take into account how this would also have external benefits to other sectors and other Sustainable Development Goals (SDGs).

## 2. Methodology for Calculating Lost Potential and Secured Potential

### 2.1 Data Overview

**Countries and Groupings:** the model is run for 217 countries, using the World Bank country codes and names as the key. Income level groups are based on the World Bank classifications and are updated approximately once a year.

The four income groups are: Low Income (LIC), Lower Middle Income (LMIC), Upper Middle Income (UMIC) and High Income (HIC). Regional groups are also based on the World Bank and UN classifications. The seven World Bank groups used are: East Asia and Pacific (EAP), Europe and Central Asia (ECA), Latin America and Caribbean (LAC), Middle East and North Africa (MENA), North America (NA), South Asia (SA) and Sub-Saharan Africa (SSA). The nine UN groups used are: East Asia and Pacific (EAP), Eastern Europe and Central Asia (EECA), Western Europe (WE), Latin America and Caribbean (LAC), Middle East and North Africa (MENA), North America (NA), South Asia (SA), Eastern and Southern Africa (ESA) and West and Central Africa (WCA).

**Population of 10 year olds:** This data is from the United Nations Population estimates at the national level, by gender and by total, estimated for 2015-2020 and using the medium variant forecast for 2021-2030. This is updated approximately every two to three years.

**Learning Poverty Rate:** This data is from the World Bank at the national level, by gender and by total. This is updated approximately once a year. This combines:

- a. Primary school age children out of school (%), by gender and by total
- b. Pupils below minimum reading proficiency at end of primary (%), by gender and by total
- c. Where (a) are assumed to be learning poor, and (b) applies to the remainder, such that learning poverty = a + ((1-a)\*b)

The analysis will be updated on an annual basis to incorporate any changes in the underlying data.

## 2.2 Learning Poverty Data

Learning poverty rate data is available, by gender or by total, for 59% of countries. However, it is more commonly available for the largest countries, meaning that this data is available for 81% of the population of 10 year olds. When we split this by gender, the share falls noticeably. This is shown in Table 1.

Table 1: Learning Poverty Data

	Countries	Population of 10 year olds
Data available	59%	81%
Data split by gender	48%	33%

Learning poverty rates by gender are used where possible. If learning poverty rates are available for the total, but not the gender, the total rate for this country is used, which assumes gender parity.

For countries where no learning poverty rate data are reported, we use a weighted mean of the countries within the same income group *and* geographic region, weighted by the population of 10 year olds, on the proviso that this was averaging at least three data points. Otherwise, only the income group average was used.

## 2.3 Lost Potential

“Lost Potential” is presented on a daily basis i.e. how many children are turning 10 each day but are not able to read and understand a simple story. This is the same as the World Bank’s learning poverty measure.

This is calculated for each country by:

$$DLP = (LP * 100) * \frac{pop_{10yo}}{365}$$

Where:

- DLP = Daily Lost Potential
- LP = Learning Poverty rate
- pop\_10yo = Number of 10 year olds
- 365 = Number of days in a year.

We calculate this flow measure in terms of the number of children for each country, and then sum this up into global, income group and regional totals.

## 2.4 Secured Potential

Potential Secured presents the inverse of Lost Potential, that is how many children are turning 10 each day and are able to read and understand a simple story. This is the same as the inverse of the World Bank’s learning poverty measure.

This is calculated for each country by:

$$DSP = ((1 - LP) * 100) * \frac{pop_{10yo}}{365}$$

Where:

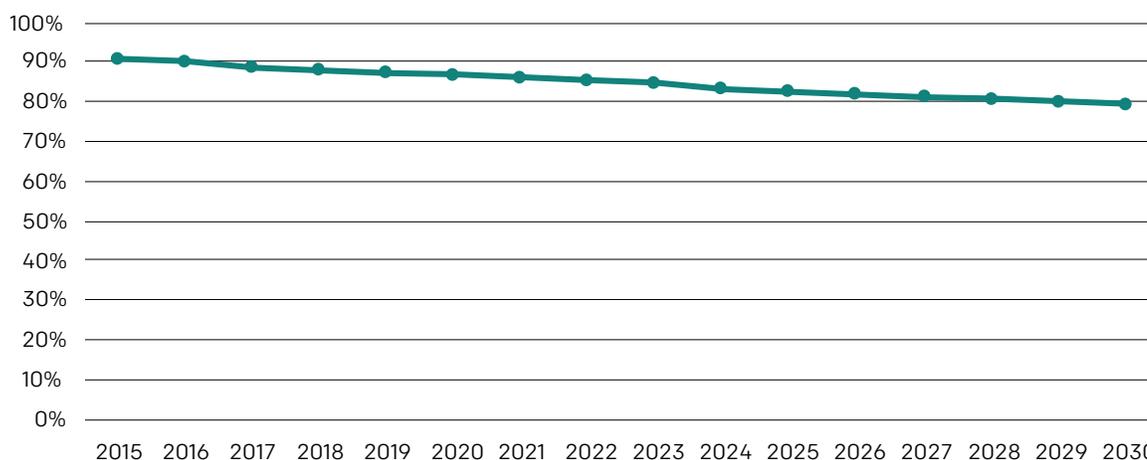
DSP = Daily Secured Potential

## 3. Scenarios Methodology

### 3.1 Pre-COVID-19 Modeling

We use the business as usual scenario, in which we model a linear change over time between the initial learning poverty for each country in 2015, and a 0.68 percentage point reduction in learning poverty each year (to a floor of 0%) based on the current rate of progress reported in the World Bank paper.<sup>2</sup> This is shown in Figure 1.

Figure 1: Visual example of pre-COVID-19 scenario learning poverty rates



### 3.2 Post-COVID-19 Modeling

We estimate the impact of COVID-19 on the LPT flow measures using the World Bank research which estimates the effects of COVID-19 on the learning poverty rates across income groups and regions.<sup>3</sup>

Table 2 shows the pessimistic scenario of the estimate of the impact of COVID-19 on learning poverty rates across income groups.

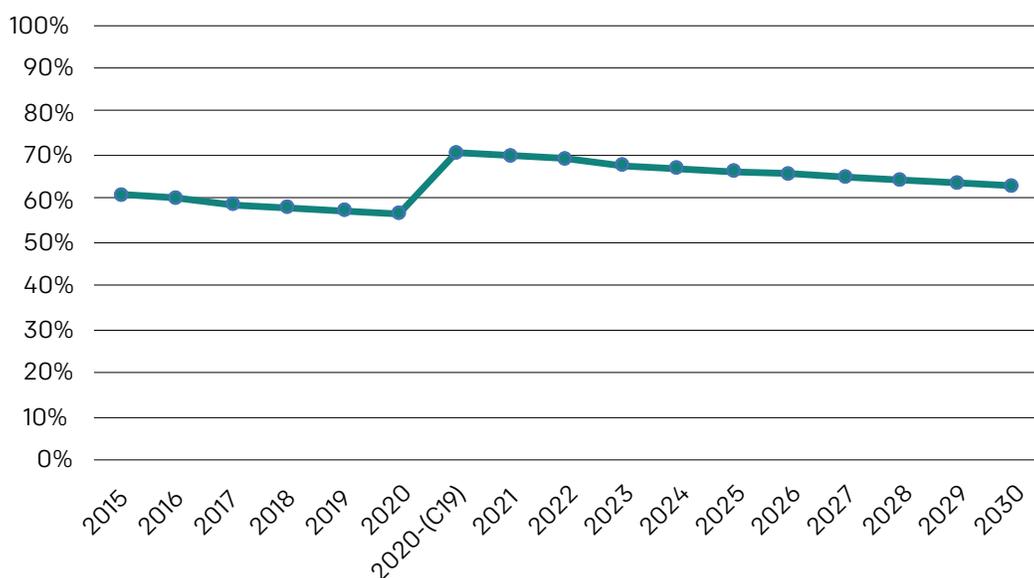
Table 2: Impact of COVID-19 on learning poverty rates by income group (percentage points)

	Pessimistic Estimate of Covid Impact
Low Income	2.9
Lower Middle Income	13.6
Upper Middle Income	5.8
High Income	4.4

Azevedo notes that “the countries that had the highest levels of learning poverty before COVID-19 might have the smallest absolute and relative increases in learning poverty, reflecting how great the learning crisis was in those countries before the pandemic.” In other words, where learning is already low in schools, the effect of school closures are likely to be smaller. The impact therefore appears greatest in LMICs where some learning was occurring in schools, but where there is less ability for mitigation and remediation during and after closures. In line with the World Bank publication<sup>4</sup> detailing this analysis, we use the pessimistic scenario for calculations of the impact of COVID-19.

The numbers of children with lost potential uses the business as usual post-COVID-19 scenario where the learning poverty rate had begun to fall at 0.68 percentage points per year, the COVID-19 impact is added in 2020 and subsequently continues to fall by 0.68 percentage points per year. This example is visualised in Figure 2.

Figure 2: Visual example of post-COVID-19 scenario learning poverty rates



## 4. Methodology for Returns to Financing on Lost Potential

### 4.1 Financing Overview

The purpose of the LPT is to drive a sense of urgency around the global learning crisis. In addition to this urgency, it is important to facilitate agency and engagement of stakeholders through highlighting how, one pathway to changing this situation, is through increased education financing that is spent efficiently and equitably.

In this analysis, we model the additional cost of bringing a child from lost potential to potential secured at a system level. We then consider how this changes with efficient spending and targeting.

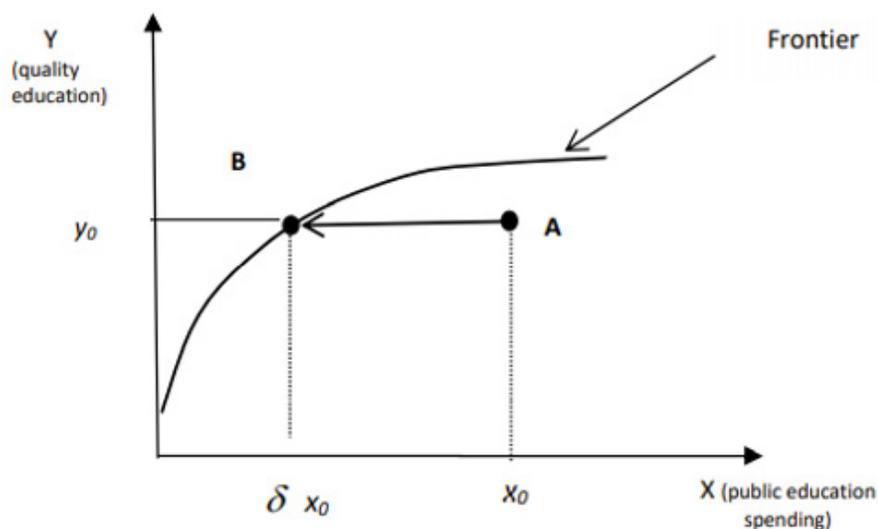
We build on the analysis of Al-Samarrai, Cerdan-Infantes and Lehe (2019), and in particular Miningou (2019), who used a stochastic frontier model to estimate the relationship between spending and LAYS.<sup>5,6</sup>

We adapt this model to estimate the relationship between spending and learning poverty as the basis of the lost potential measurement. We use the inverse of learning poverty, the share of children who can read and understand a simple story by the age of 10, which we refer to as the adjusted proficiency.

In adapting to learning poverty, we also adjust the period of time as LAYS is measured between the ages of 4-17, whilst learning poverty is concerned with the period until a child turns 10. Therefore, we focus the data relative to the average of the preceding 10 years of the learning poverty observation, and particularly towards the education spending at primary.

The stochastic frontier model uses an estimation based on the construction of a production frontier that gives the maximum outputs (quality education, or in our case the proportion of children that can read and understand a simple story by age 10) that can be achieved given the quantity of inputs (public education spending). This is visualised in Figure 3. The closer the country is to the frontier, the more efficiently they achieve the outputs. The model allows us to estimate the additional public spending required at the system level for an additional child to be able to read and understand a simple story by age 10.

Figure 3: Illustration of the frontier approach



Source: Miningou, 2019

The equations and more details on the stochastic frontier model method is presented in Appendix A as described by Miningou (2019).

## 4.2 Data and Descriptive Statistics

As with the existing literature on education production functions, we assume that countries use public expenditure on education as inputs with quality education as outputs. In this case we measure quality education as the share of children being able to read and understand a simple story by the age of 10.

### Input Variable

Our input variable is the total public spend on primary education per primary school aged child. This includes both government and Official Development Assistance (ODA) spending and is comprised of three data sources.

Firstly, data from UNESCO Institute of Statistics (UIS) is used on 'Government expenditure on primary education' in Purchasing Power Parity (PPP) dollars. This is in constant 2017 PPP dollars.

Secondly, data from OECD's Development Assistance Committee (DAC) database is used from 'all official donors' (which includes both DAC and non-DAC countries as well as multilateral institutions). This records for each recipient country, the annual ODA disbursed that was specifically assigned to the Detailed Sector Code of 'Primary Education (11220)'. This is available in both current United States Dollars (USD) and constant 2018 USD. The constant 2018 USD is converted into 2017 PPP dollars by first converting into constant 2017 USD and then converting to PPP using the 'Price level ratio of PPP conversion factor' from the World Bank.<sup>7</sup> For countries that received no annual ODA, this is set to zero.

Thirdly, data from UIS is used on 'School age population, primary education, both sexes (number)'.<sup>8</sup> For the year that learning poverty data is available, the mean of each of these three variables over that year and the

preceding nine years (for a total of ten years' time period) is calculated to take into account that outcomes at aged 10 are the result of cumulative spending in the preceding years. The government expenditure on primary education in constant 2017 PPP dollars is added to the ODA on primary education in 2017 PPP dollars. This total public spend on primary education is divided by the school age population to calculate the total public spend on primary education per primary school aged child.

Household expenditure can also be a significant component of spending on education, particularly at lower income groups. However, this was not included in this analysis due to the lack of available data.

### *Output Variable*

The key measure behind the LPT is the learning poverty rate. As the stochastic frontier model is typically considered in terms of a positive relationship, i.e. where an increase in inputs is associated with an increase in outputs, we use the inverse of learning poverty as our output variable. This is the share of children who can read and understand a simple story by the age of 10, which we refer to as the adjusted proficiency, and is equal to  $1-(LP\%)$ . This measure takes into account both access and learning.

In some cases, the learning poverty rate, and therefore also the adjusted proficiency rate, is available for a country for more than one year. For robustness, we also compare the effects of using the weighted average for each observation, where the weight is equal to  $1/(\text{number of observations available for that country})$ . The weighted results are similar and presented in Appendix B. The main results presented below are using only the latest observations.

### *Additional variables investigated*

In line with the existing literature, a number of additional variables were considered for control variables. This includes income groups using the World Bank definitions and geographic region groups using the World Bank and United Nations definitions. In addition, the mean over the ten years' time period as described above was calculated for the following variables:

- The share of ODA in total public spending on primary – calculated using the data described in the input variable section above, with ODA to primary divided by total public spending on primary.
- The primary completion rate – from the World Bank Data Bank compilation of UIS data
- The student to trained teacher ratio – from UIS data
- The 'percentage of students in primary education who have their first or home language as language of instruction' – from UIS data
- The unemployment rate – from the World Bank Data Bank compilation of International Labour Organisation data, including the rate for the total population and a specific rate for only the population with basic education
- The gross enrolment rate of pre-primary - from the World Bank Data Bank compilation of UIS data
- The primary out of school rate - from the World Bank Data Bank compilation of UIS data
- The child labour rate – from the World Bank Data Bank compilation of International Labour Organisation data

These variables were considered for inclusion into the stochastic frontier model to better identify the potential sources of inefficiency in the education spending, as described in equation 2 of Appendix A. However, with the exception of the share of ODA in total public spending on primary, they did not improve the stability of the model and the results we present here do not include these. For the share of ODA in total public spending on primary, we refer to this as the 'ODA share' and include this in some of the specifications shown in Section 4.3 and 4.4.

## Descriptive Statistics

Table 3 presents the descriptive statistics of these variables, for the latest available observation per country.

Table 3: Descriptive statistics

Variable	Obs	Mean	St Dev.	Min	Max
<b>Input Variable</b>					
Total public expenditure on primary per primary school aged child	144	3,388.67	3,885.53	34.51	16,240.38
<b>Output Variable</b>					
Adjusted Proficiency	114	61.69	32.02	1.28	98.36
<b>Additional Variables</b>					
ODA Share	145	0.05	0.09	0.00	0.53
Primary Completion Rate	183	88.87	17.87	27.44	121.66
Student to Trained Teacher Ratio	126	37.38	29.27	7.68	234.78
Same Language	71	68.30	33.12	0.41	99.22
Unemployment Rate (Total)	187	8.01	5.78	0.34	34.47
Unemployment Rate (Basic Education)	152	10.28	8.45	0.12	47.89
Pre-Primary Gross Enrolment Rate	187	60.01	36.77	1.09	197.45
Primary Out Of School Rate	186	8.10	11.20	0.01	58.93
Child Labour Rate	75	21.03	15.33	1.00	63.92

No values are replaced or imputed for missing input or output variables. Where data was missing for the additional variables (with the exception of ODA share as this was never missing whilst the input variable data exists) this was replaced with the weighted mean of countries within the same income group *and* geographic region, weighted by the school age population, on the proviso that this was averaging at least three data points, otherwise only the income group average was used.

The variation in these key variables is worth unpacking in greater detail across income groups. This is presented in Table 4 for the input and output variables.

Table 4: Variation in key variables

Variable	Obs	Mean	St Dev.	Min	Max
<b>Low Income</b>					
Total public expenditure on primary per primary school aged child	20	162.65	71.86	34.51	335.59
Adjusted Proficiency	12	5.33	1.28	17.20	8.73
<b>Lower Middle Income</b>					
Total public expenditure on primary per primary school aged child	37	627.30	365.86	174.46	1,725.32
Adjusted Proficiency	24	39.85	24.71	5.07	98.33
<b>Upper Middle Income</b>					
Total public expenditure on primary per primary school aged child	34	2,071.36	1,323.73	623.13	7,267.64
Adjusted Proficiency	29	60.51	23.39	19.26	97.82
<b>High Income</b>					
Total public expenditure on primary per primary school aged child	53	7,378.85	3,677.01	1,387.46	16,240.38
Adjusted Proficiency	49	86.04	15.80	33.37	98.36

### 4.3 Stochastic Frontier Model results

The greatest constraint on the number of observations is the extent of the learning poverty, and therefore adjusted proficiency data. This is available for 114 countries, although as described in Section 2 these do contain a large majority of the world's population. Of these 114 countries, 95 also have spending data within the time period. The input and output variables are logged, and the marginal effects capture elasticities.

In the first instance, we run the stochastic frontier model on these countries, allowing only one observation per country using the most recent data. This is shown in Table 5. We present the results both without including any explanatory factors of the efficiency and including ODA share as an explanatory factor. We also vary the frontier variables, in the first instance only including total public expenditure on primary per primary school aged child, and its squared term. To this we also add regional dummy variables as controls. This gives four specifications which we present here:

- Model 1a – Without regional dummies, without ODA share
- Model 1b – Without regional dummies, with ODA share
- Model 2a – With regional dummies, without ODA share
- Model 2b – With regional dummies, with ODA share

Table 5: Results of the stochastic frontier estimations

	Model 1a	Model 1b	Model 2a	Model 2b
<b>Frontier</b>				
ln_expPC_p_avg	0.987*** (0.253)	0.833** (0.260)	0.839*** (0.000)	0.905*** (0.119)
sq_ln_expPC_p_avg	-0.043* (0.018)	-0.038* (0.018)	-0.040*** (0.000)	-0.047*** (0.009)
world_region_eap			0.580*** (0.000)	0.562*** (0.092)
world_region_eca			0.364*** (0.000)	0.372*** (0.100)
world_region_lac			0.041*** (0.000)	0.013 (0.079)
world_region_mena			0.018*** (0.000)	0.101 (0.090)
world_region_na			0.047*** (0.000)	0.325 (0.177)
world_region_sa			0.552*** (0.000)	0.618*** (0.133)
world_region_ssa			-0.000 (.)	0.000 (.)
Constant	-0.510 (0.897)	0.354 (0.896)	0.171*** (0.000)	0.038 (0.367)
<b>Usigma</b>				
odashare_p_avg		15.888*** (3.855)		20.193*** (3.929)
Constant	-0.528 (0.270)	-1.251*** (0.273)	-0.550*** (0.145)	-1.775*** (0.260)
<b>Vsigma</b>				
Constant	-2.925*** (0.583)	-3.627*** (0.541)	-35.657 (320.748)	-5.318*** (1.029)
Observations	95	95	95	95
Wald chi2 (2) =	61.35	64.37	1.03e+11	378.76
Prob > chi2 =	0.0000	0.0000	0.0000	0.0000
Average efficiency score:	0.618	0.635	0.646	0.677

\*Significant at 5%; \*\*significant at 1%; \*\*\*significant at 0.1%

The input and output variables are logged, and the marginal effects capture elasticities

In each of these specifications, the spend term is significant at least at the 1% level and the squared spend term is significant at least at the 5% level. The directions and coefficients of the spend term and squared spend term are fairly consistent across specifications.

We also test the robustness by allowing more than one observation per country and weighting by the number of observations per country. This increases the number of observations to 185. This is shown in Appendix B.

We also model this without the squared spend term. The logged nature of the variable means that the non-linear relationship still exists but to a lesser extent, and these results presented in Appendix B.

#### 4.4 Outcomes and Bootstrap Model Results

The stochastic frontier model results are using logged variables. The marginal effects capture elasticities which we model to estimate the outcomes in terms of the additional public spend required at the system level for each additional child to read and understand a simple story by the age of 10.

We use a similar methodology as the Miningou (2019) outputs contribute to the GPE Methodology in producing the ‘cost per additional year of quality schooling’. In our case, we start from the stochastic frontier model based on the equation (1) in Appendix 1, simplified here as equation (A):

$$\text{Ln(Adjusted_Proficiency)} = \beta_0 + \beta_1(\text{Ln(Spend_Per_Child)}) + \beta_2(\text{Ln(Spend_Per_Child)}^2) + \dots \quad (\text{A})$$

As stated in Table 5, the marginal effects capture elasticities. We therefore follow the steps in Table 6 to get from the stochastic frontier model results to the stated outcome.

Table 6: Calculating outcomes from the regression outputs

	Calculation	Example for Low Income – Model 1a	Example for Low Income – Model 2a
Adjusted proficiency	(i) = from data	8.90	8.90
Spend per child	(ii) = from data	173.61	173.61
Taking the first differential of the spend, and inputting the log of the spend	(iii) = $\beta_1 + (2 * \beta_2 * \text{Ln}(\text{ii}))$	0.54	0.42
The percentage increase needed to increase one percentage point of adjusted proficiency	(iv) = $1/(\text{i})$	0.11	0.11
Dividing the percentage increase by the output of the first differential	(v) = $(\text{iv})/(\text{iii})$	0.21	0.27
Additional cost per child needed to adjust proficiency by one percentage point	(vi) = $(\text{v}) * (\text{ii})$	36.04	46.16
Total Children turning ten each year	(vii) = from data	17,729,774	17,729,774
Additional total spending if that cost was spent for every child	(viii) = $((\text{ii}) + (\text{vi})) * (\text{vii}) - (\text{ii}) * (\text{vii}) = (\text{vi}) * (\text{vii})$	638,928,929	818,396,439
Additional number of children with adjusted proficiency if that spending was added	(ix) = $((\text{i}) + 1) * (\text{vii}) - (\text{i}) * (\text{vii}) = (\text{vii}) * 1\%$	177,298	177,298
Additional cost per child with adjusted proficiency	(x) = $(\text{viii})/(\text{ix})$	3,603.71	4,615.94

Whilst this can be calculated at this stage for each income group, the relatively low sample size and the sensitivity of the stochastic frontier model led us to develop a model to bootstrap the estimates received from this by income group. Bootstrapping of the sample with replacement and repeating this 1,000 times for each specification we receive the following results presented in Table 7.

Table 7: Bootstrapped estimates for 1,000 repetitions

	LICs	LMICs	UMICs	HICs
Model 1a	3,545**	4,529***	11,185	37,404
Model 1b	4,376	5,733	14,681	53,301
Model 2a	4,522**	6,084***	16,220**	65,414
Model 2b	4,606*	6,649***	20,038*	127,050

\*Significant at 5%; \*\*significant at 1%; \*\*\*significant at 0.1%

Note in model 2a and 2b, one or more parameters could not be estimated in 18 and 19 bootstrap replicates respectively. This occurs on the occasions where the bootstrapped sample happens to not include any of the three countries in the North America region. This means the parameters cannot be estimated for that sample but these few occurrences do not affect estimated values. Standard-error estimates include only complete replications.

We can see that the significance of these bootstrapped estimates are stronger for LICs, LMICs, and to a lesser extent UMICs. In contrast, the greater variety of both spending and outcomes in HICs reduces the significance of estimates for HICs. This is also the case if all high income MENA countries are moved into upper middle income.

Similarly, we repeat this for when we allow the multiple observations per country weighted, and without the squared term and present this in Appendix B. In the case of removing the squared term, this decreases the cost for HICs and these bootstrap estimates are significant in this case. However, the model presented with the squared term still appears a greater fit with our perceptions and the existing literature.

The model used for the LPT and the favoured model is model 2a, which includes the regional dummies and without any explanatory factors of the efficiency, where the spend term and squared spend term of the stochastic frontier model are significant at the 1% level, and the bootstrapped estimates over 1,000 replications are significant at the 1% level for LICs, LMICs, and UMICs.

This provides the additional public spend required at the system level for each additional child to read and understand a simple story by the age of 10 of \$4,522 for LICs, \$6,084 for LMICs, \$16,220 for UMICs and \$65,414 for HICs. However, this is modeling a specific view of spending at the system level.

This does not assume that the additional financing is spent efficiently, nor that this financing is specifically targeted towards those that do not have basic literacy skills at age 10. Moreover, this is also calculated based on the binary nature of either having foundational literacy skills or not by age 10, and does not consider how far away each child is from this threshold at age 10. For example, we know that children may instead acquire basic literacy skills by age 11 or age 12, and that in reality the situation is not as binary as is currently modelled. To take this into account, we also make adjustments for these factors, which is explained in the following sub-sections.

#### 4.5 Efficiency of Spending

The stochastic frontier model described above corrects for the bias of the parameters caused by inefficiencies, but the output does not assume full efficiency. Instead, it has estimated the effect of spending on learning proficiency, given the actual inefficiencies in the system.

The average efficiency of spend for each income group can be calculated and used to estimate the cost for each additional child to read and understand a simple story by the age of 10 if the financing is spent at full efficiency.

Using the same method as Miningou (2019), an efficiency score is derived for each country that reflects the ability of the country to translate their inputs into outputs. The exact mathematical formula is presented in Appendix A (Equation 3). We use the estimations presented in Table 5 for the preferred Model 2a. The efficiency score varies between 0 and 1 and captures the relative capacity of countries to make effective use of the financial resources provided to the education sector. It is important to notice that this is a relative measurement of efficiency and the results may vary if there are changes in the sample of countries considered or in the variables included in the analysis.

This is shown disaggregated by income group in Table 8. Across all countries, the average efficiency is 65%. This means that globally, the number of children that can read and understand a simple story by age 10 would have been achieved with 35% less spending if all countries were efficient. In other terms, making all countries as efficient as the most efficient countries in the sample could have saved, on average 35% of the public expenditure on education.

Table 8: The effect of financing on lost potential at full efficiency of spending

	Formula	LICs	LMICs	UMICs	HICs
Unit cost with inefficiencies	(xi) = Model 2a From Table 5	4,522	6,084	16,220	65,414
Average efficiency score	(xii) = Output from Model 2a	0.340	0.590	0.675	0.740
Unit cost if spending at full efficiency	(xiii) = (xi) * (xii)	1,539	3,592	10,942	48,399

#### 4.6 Targeting of Spending and the Learning Poverty Gap

Similarly, it is important to also take into account learning poverty severity and the targeting of spending. World Bank research<sup>9</sup> highlights that one weakness of the learning poverty rate is the binary nature of a child being able to read and understand a simple story by age 10 compared to a child not being able to do so at this age. This does not take into account the distance that children are from this threshold.

Following the logic of cash poverty, which uses both the headcount index and the poverty gap index to take into account both the rate and the severity of cash poverty, the World Bank research develops the measure of learning poverty severity to go alongside the learning poverty rate. This paper distinguishes between learning deprivation (only the children in school) and learning poverty (that includes out of school children) and defines the Learning Deprivation Gap as the average distance of a learning deprived child to the minimum reading proficiency level, with the same principle applying to learning poverty severity.

Therefore, using this learning poverty gap allows us to meet children at the average distance they are from being able to read and understand a simple story by age 10. This allows for a more nuanced understanding of the cost of bringing a child out of learning poverty. The outputs presented in section 4.4 and 4.5 consider only the binary difference between a child being able to read and understand a simple story by age 10 and not being able to do so by age 10. Incorporating the learning poverty gap takes into account the learning trajectory and how far children are from being able to read and understand a simple story by age 10.

The World Bank research presents this learning poverty gap across both the total population and specifically among those that are learning poor. In line with Assumption 3 in the Global Partnership for Education's (GPE's) *Methodology for replenishment indicators*, we also assume that spending will be targeted at those that are learning poor.<sup>10</sup> The estimates in Table 9 are what was used for the LPT.

**Table 9:** The effect of financing on lost potential at full efficiency of spending and targeting the learning poor at their distance from being able to read and understand a simple story by age 10

	Formula	LICs	LMICs	UMICs	HICs
Unit cost if spending at full efficiency	(xiii) = From Table 6	1,539	3,592	10,942	48,399
Learning poverty gap (among the learning poor)	(xiv) = From Azevedo (2020)	45.1%	26.0%	30.0%	46.7%
Unit cost if spending at full efficiency targeting the learning poor	(xv) = (xiii) * (xiv)	694	934	3,283	22,602

## 4.7 Financing Conclusion

This analysis provides the additional public spend required for each additional child to read and understand a simple story by the age of 10 if spent efficiently and targeted to the average learning poor child. The additional spend per child to bring a child from lost potential to secured potential is \$694 for LICs, \$934 for LMICs, \$3,283 for UMICs and \$22,602 for HICs.

For the purposes of the LPT, we assume that these costs are spent over 6 years, and instead use the spending required by year. As such, we divide the figures above by 6 to get the spending required per year to set a child on the path out of learning poverty. The per year unit costs are \$116 for LICs, \$156 for LMICs, \$547 for UMICs and \$3,767 for HICs.

This methodology makes the following assumptions as described throughout Section 4:

1. Spending is additional
2. Spending is fully efficient
3. Spending is targeted only to children that are currently learning poor
4. Spending is targeted to children below 10
5. Spending brings the child from the average distance of learning poor to reach proficiency

There is significant overlap between the assumptions used here and those in the GPE methodology. Assumptions 1 and 3 are in both methodologies. Assumption 4 serves as an extension to assumption 3, in assuming that the spending is focused towards relevant beneficiaries. Assumption 2 varies slightly in that we assume this spending is efficient, whereas the GPE methodology assumes that their spending brings about additional efficiency benefits. Similarly, Assumption 5 varies slightly in that we target to the average learning poor child, whereas the GPE methodology considers the average child.

## 5. Bounds on changing financing

For the functionality of the LPT, it is important to restrict upper bounds on the changes in financing that can be modelled. As part of this, it is important to differentiate between one-off financing and annual financing.

### 5.1 Additional financing

The key consideration for setting these bounds is that these are static estimations of a dynamic situation. In other words, this does not currently take into account the myriad changes that would go on by the time a LIC had reduced learning poverty to the extent of HICs now. By the time these changes had occurred, the costs and context of that country would likely have changed significantly.

To calculate relevant bounds for the additions in financing, we use the unit cost set out in section 4.7. and compare this to the number of children it would require to gain secured potential on an annual basis to change the learning poverty rate for that income group to the next income group (i.e. for how many children with secured potential before the low income rate of learning poverty becomes equal to the current lower middle income rate of learning poverty). For high income this is compared to ending learning poverty.

Table 10: Estimating bounds for additions in annual financing

	Formula	LICs	LMICs	UMICs	HICs
Learning Poverty rate for countries in financing sample	(xvi) = From Data	91.1%	61.4%	38.9%	12.9%
Learning Poverty rate for countries in financing sample of next income group	(xvii) = From Data	61.4%	38.9%	12.9%	0.0%
Unit cost if spending at full efficiency targeting the learning poor	(xv) = From Table 7	694	934	3,283	22,602
Total Children turning 10 (2020)	(xviii) = From Data	17,729,774	58,348,937	40,448,177	13,590,964
Annual change in financing to get to the next income groups Learning Poverty Rate	(xix) = (((xvii) – (xvi)) * (xviii)) * (xv)	3,652,430,143	12,292,629,685	34,470,301,250	39,629,490,434

The annual change in financing to get to the next income group’s learning poverty rate can be used as the upper bound of financing for any annual change in spending. For simplicity, for the LPT, the upper bound for all income groups will be the lowest upper bound of all income groups, in this case for LICs of \$3.5 billion. The LPT also includes financing over 10 years (2021-2030), so these annualized bounds are multiplied by 10.

## 6. External benefits to other SDGs

In addition to estimating how education financing can affect the number of children experiencing lost potential, the purpose of the LPT can be further strengthened through highlighting the potential externalities from education as well. This will focus on the benefits of additional education financing.

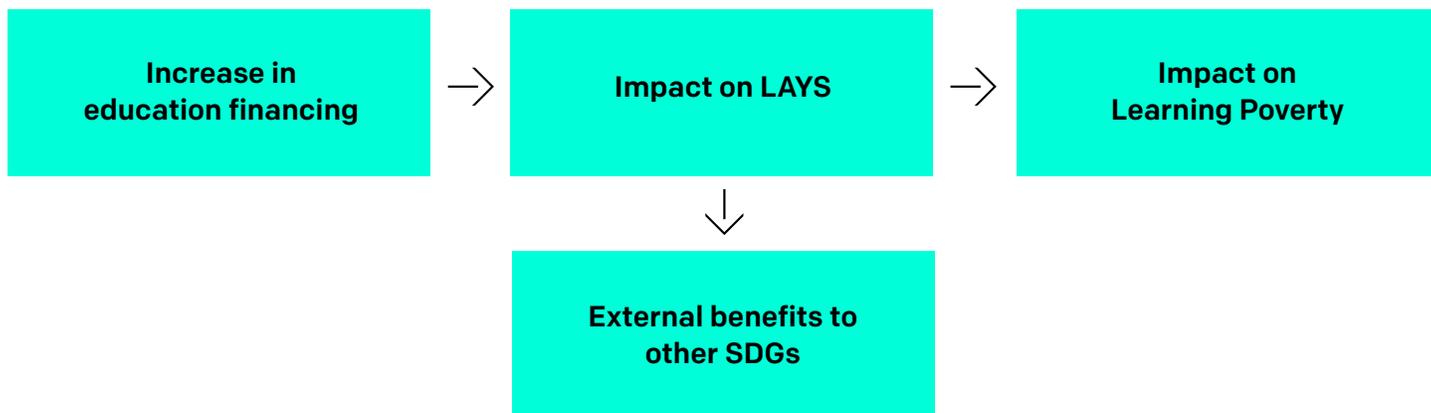
Here, we use the methodology from the GPE’s ‘Methodology for the Replenishment Indicators’ which incorporates recent research from the World Bank (Miningou, 2019) and others.<sup>11</sup> This draws on the well-established literature looking at the external benefits to increases in ‘years of schooling’ whilst also bringing in learning through the LAYS measure which incorporates the effects of both access and learning that are also at the basis of the LPT.

These external benefits are estimated through the impact of LAYS rather than the impact of learning poverty specifically, primarily due to this greater availability of literature. Whilst it would be possible to run similarly specified regressions focused on learning poverty, this would depart from the peer-reviewed and approved methodologies that are available.

The key limitation of this analysis however is that the LAYS indicator looks at a larger age range than the learning poverty / lost potential measures. As previously highlighted, and set out in more detail below, LAYS uses the quality of education between the age range of 4-17, with the quality aspect commonly relating to exams at around age 15. On the other hand, learning poverty and lost potential data focus on the access and learning achieved by age 10 and the end of primary. We have stated the assumption in Section 4 that the education financing is focused on those below aged 10, but this limitation still remains.

Nevertheless, for the user inputting a particular amount, duration, and income group focus, this will show the impact this has on lost potential, learning poverty and the external benefits to other SDGs described in this section. This is visualised in Figure 4.

Figure 4: The flow of education financing impacts



### 6.1 Cost of one additional year of LAYS

The relationship between LAYS and education financing was first investigated by Al-Samarrai, Cerdan-Infantes and Lehe (2019). This was taken further by Miningou (2019) who approximated the following costs of one additional year of quality education in US Dollars (USD) as in Table 11. In line with the similar analysis in Section 4, we again assume that this spending is at full efficiency. The average efficiency scores by income group are different to those in Section 4 as here these are calculated for LAYS as the outcome variable, from Miningou (2019), instead of adjusted proficiency – where the stricter binary nature of the measure for each child at age 10 results in lower efficiency scores.

Table 11: Cost of one additional year of LAYS (USD)

	Formula	LICs	LMICs	UMICs	HICs
Cost of one additional year of quality education	(a1) = From source below	118.52	397.95	1885.74	8084.60
Average efficiency by income group	(a2) = From source below	0.752	0.852	0.876	0.973
Cost of one additional year of quality education at full efficiency	(a) = (a1) * (a2)	89.13	339.05	1651.91	7866.32

Source: Cost calculated by Miningou, previously Low Income and Lower Middle Income Countries were presented in GPE (2020),<sup>12</sup> whilst Upper Middle Income and High Income Countries added in consultation with Miningou. Calculations include government spending and Official Development Assistance. Efficiency calculated by Miningou, presented in Miningou (2019).<sup>13</sup>

The number of children who receive an increase in LAYS from an example amount of education financing of \$1 billion per income group is shown in Table 12.

Table 12: Change in LAYS based on example financing

	Formula	LICs	LMICs	UMICs	HICs
Additional children reaching 1 additional LAYS	(b) = 1bn / (a)	11,219,940	2,949,388	605,361	127,124

### 6.2 GDP (SDG 8)

The GPE methodology builds on the research of Hanushek and Woessman (2007),<sup>14</sup> which estimates a causal relationship between years of schooling and GDP. The GPE methodology takes this further and estimates the impact of LAYS and GDP, finding that one additional unit of LAYS is associated with 0.894 percentage points increase in the GDP per capita growth rate.

Using this result and the number of individuals whose LAYS have increased as a result of the spending, we can estimate the potential increase in GDP per capita associated with this, weighted over the population. Comparing this to the average GDP per capita growth rate for each income group over the past ten years, and the latest GDP per capita, we can project forward the increase in GDP per capita growth rates associated with this increase in LAYS.

The GPE paper assumes that this benefit is felt over 47 years, with an average individual working from 18 to 65. As we are assuming that our investments are made before the child turns 10, we assume a gap of eight years before the increase in GDP per capita growth rates begins.

The present value of this future income generated is estimated using a discount rate. In the GPE methodology, they use a discount rate of 1.69% based on average inflation rates of their targeted countries. In our methodology, we instead use a more conservative discount rate of 5%. This change also results in a slightly more conservative estimate of the overall returns of financing, albeit still in line with other estimates in the literature given the discount rate used. This is shown in Table 13.

Table 13: External benefits to GDP (USD)

	Formula	LICs	LMICs	UMICs	HICs
Additional children reaching 1 additional LAYS	(b) = From Table 8	11,219,940	2,949,388	605,361	127,124
Population (2020)	(c) = From data	668,454,965	2,913,363,391	2,855,862,780	1,235,852,838
Increase in LAYS weighted over the population	(d) = (b) / (c)	0.01678	0.00101	0.00021	0.00010
Percentage point increase in GDP per capita growth rate due to the increase in LAYS	(e) = 0.894 * (d)	0.01501	0.00091	0.00019	0.00009
GDP per capita 2020 current USD	(f) = From data	810.10	2,174.44	9,036.77	44,617.48
GDP per capita growth rate (10 year average)	(g) = From data	2.15%	1.81%	2.06%	1.45%
Net Present Value of increase in GDP per capita growth rate	(h) = Calculated as described above	7.21	0.98	0.86	1.77
Benefit cost ratio	(i) = (h) divided by NPV of cost in year zero	4.82	2.85	2.45	2.19
Increase in GDP	(j) = (h) * (c) = (i) * 1bn	4,817,080,518	2,852,238,846	2,450,558,926	2,192,277,387

### 6.3 Extreme Poverty (SDG 1)

From this impact on the GDP, we can also estimate the effect on income poverty (as opposed to learning poverty). We consider the extreme poverty gap of \$1.90 per day for all income groups. Using the same data source as the GPE methodology, we can estimate the share of this that supports household consumption for each income group. Assuming that the targeting of those in learning poverty spreads this across both the income poor and non-poor, we can estimate the amount received by those in income poverty. In the same manner as the GPE methodology, comparing this to the average poverty gap within those countries, over the period of time considered, we can estimate the number of people that could reach the extreme poverty line and become non-poor. This is shown in Table 14.

Table 14: External benefits to extreme poverty reduction (USD)

	Formula	LICs	LMICs	UMICs	HICs
Increase in GDP	(j) = From table 9	4,817,080,518	2,852,238,846	2,450,558,926	2,192,277,387
Households share of final consumption expenditure	(k) = From Data	73.00%	66.00%	51.00%	59.00%
Incidence of Poverty (extreme poverty line)	(l) = From Data	45.50%	16.90%	1.60%	0.60%
Poor households increase in consumption	(m) = (j)*(k)*(l)	1,599,993,294	318,138,721	19,996,561	7,760,662
Poverty Gap at 1.90 USD per day (extreme poverty line)	(n) From data	18.00%	4.30%	0.40%	0.40%
Estimated number lifted from extreme poverty	$o = (m) / ((n) * 1.90 * 365 * (47 + 8))$	233,043	193,972	131,065	50,866

#### 6.4 Government Revenue (SDG 17)

From this impact on the economic output, we can also estimate the effect this has on government revenue generation and relatedly the available public budget. Whilst acknowledging that revenue collection and spending can vary significantly between countries even within income groups, we calculate the population weighted averages for each income group using the World Bank data on 'revenue (excluding grants) as a share of GDP'.

Applying these to the economic output generated, we can estimate the increase in public revenue generated. This is shown in Table 15.

Table 15: External benefits to public revenue generation (USD)

	Formula	LICs	LMICs	UMICs	HICs
Increase in GDP	(j) From Table 9	4,817,080,518	2,852,238,846	2,450,558,926	2,192,277,387
Revenue (excl. grants) as a share of GDP	(p) From Data	12.07%	14.41%	15.45%	18.52%
Public revenue generated	(q) = (j)*(p)	581,421,619	411,007,618	378,611,354	406,009,772

#### 6.5 Health (SDG 3)

The GPE paper builds on the Education Commission's econometric specification<sup>15</sup> and applies this to more recent data. Bringing in LAYS resulted in an estimate that one additional unit of LAYS is associated with 1.55 percentage point decrease in the mortality rate. In the GPE methodology, this is multiplied by 3.3 LAYS for each individual to get to a 5.13 percentage point change. As we are only following a one unit change in LAYS, we keep the 1.55 percentage point change. Following the similar steps as the GPE methodology and applying this change in the mortality rate to the weighted impact on the total population, the number of lives saved as a result of the education financing can be calculated. This is shown in Table 16.

Table 16: External benefits to health (USD)

	Formula	LICs	LMICs	UMICs	HICs
Additional children reaching 1 additional LAYS	(b) = From Table 8	11,219,940	2,949,388	605,361	127,124
Total Population	(r) = From Data	668,454,965	2,913,363,391	2,855,862,780	1,235,852,838
Increase in survival rate due to addition LAYS weighted by population	(s) = 1.55% * (b) / (r)	0.000261	0.000016	0.000003	0.000002
Lives Saved	(t) = (s) * (r)	174,582	45,892	9,419	1,978

## 6.6 Early Marriage and Gender (SDG 5)

The GPE methodology builds on a study conducted by the World Bank<sup>16</sup> which finds that a unit increase in the average years of schooling reduces the likelihood of child marriage by 7.5 percentage points. This relationship is not adapted further to take into account further benefits from LAYS relative to average years of schooling.

Following the methodology of GPE and using this relationship without adaptation to LAYS, and assuming that girls receive half of the benefits of the education financing (and therefore half of the increase in LAYS), the number of girls prevented from early marriage can be estimated. This evidence is focused on additional years of secondary, so we assume these benefits are only felt by the share of children expected to progress to secondary, calculated using the share of children that reach the last grade of primary, and the share of those children that then progress to secondary. This is shown in Table 17 below.

Table 17: External benefits to reducing early marriage

	Formula	LICs	LMICs	UMICs	HICs
Additional children reaching 1 additional LAYS	(b) = From Table 8	11,219,940	2,949,388	605,361	127,124
Addition girls reaching 1 additional LAYS	(w) = (b)/2	5,609,970	1,474,694	302,680	63,562
Persistence to last grade of primary	(x) = From Data	49.46%	79.56%	93.72%	95.47%
Progression to secondary school	(y) = From Data	76.18%	89.23%	97.64%	97.38%
Girls prevented from early marriage	(z) = 7.5% * (w) * (x) * (y)	158,517	78,516	20,774	4,432

## 6.7 Nutrition (SDG 2)

These externalities can be developed wider than the inclusions in the GPE methodology, for example in terms of nutrition, climate change and disasters. The key drawback in these cases is typically a less developed literature. In particular, much of the evidence in these cases brings in education in terms of categorical components such as 'completed primary' and 'completed secondary' or grouped years of education such as '4-6 years' and '7-9' years rather than the preferred analysis at the margin in terms of additional years of schooling.

In the case of nutrition, Alderman and Headey<sup>17</sup> estimate the importance of parental education for child nutrition across a broad subset of countries. This relationship between the grouped years of education for both maternal and paternal education and its impact on and stunting is calculated for '4-6 years', '7-9 years', '10-12 years' and '13+ years'. Using similar logic to the GPE Methodology which increases LAYS from the average of that income group, we can estimate the change of one additional year for the relevant grouped years category. This is shown in Table 18.

Table 18: External benefits to reducing child stunting

	Formula	LICs	LMICs	UMICs	HICs
Average LAYS	(aa) = From Data	4.34	6.56	7.89	10.43
Impact of mothers education on stunting	(ab) = From literature for additional level of LAYS for ((aa)+1) minus previous level, divided by the years in that group	$=-(-0.010-0)/3 =$	$=-(-0.024-0.010)/3 =$	$=-(-0.024-0.010)/3 =$	$=-(-0.048-0.024)/3 =$
		-0.0033	-0.0047	-0.0047	-0.0080
Impact of fathers education on stunting	(ac) = From literature for additional level of LAYS for ((aa)+1) minus previous level, divided by the years in that group	$=-(-0.007-0)/3 =$	$=-(-0.019-0.007)/3 =$	$=-(-0.019-0.007)/3 =$	$=-(-0.029-0.019)/3 =$
		-0.0023	-0.0040	-0.0040	-0.0033
Additional children reaching 1 additional LAYS	(b) = From Table 8	11,219,940	2,949,388	605,361	127,124
Addition girls / boys reaching 1 additional LAYS	(w) = (b)/2	5,609,970	1,474,694	302,680	63,562
Total Fertility Rate per woman (assumed same per man)	(ad) = From Data	4.604	2.765	1.897	1.600
Reduced Stunting	(ae) = ((ab)* (ad)* (w)) + ((ac)* (ad)* (w))	146,360	35,339	4,976	1,153

## Appendix A

Extract from Miningou (2019, p.5-6) describing the equations used in the stochastic frontier model:

“Consider input  $x \in R$ , and the output  $y \in R$ . Following Battese & Coelli (1995), the frontier function is given by:

$$\text{Log}(y)_n = \text{Log}[f(x)_n] + u_n - \lambda_n$$

with n an indicator for countries.

The function  $f(\cdot)$  approximates the maximum educational outcomes that can be achieved given different levels of expenditure on education. Deviations from the estimated production frontier are attributable to inefficiency ( $\lambda_n$ ), as well as “noise” ( $u_n$ ).  $\lambda_n$  captures the inefficiency with which education expenditure is translated into educational outcomes in country n.  $u_n$  is normally distributed while  $\lambda_n$  follows a half-normal distribution.

The efficiency in the use of public financial resources allocated to the education sector may be influenced by some environmental factors (countries’ income level, institutional capacity, etc.). Derivation of the efficiency indicator should take into consideration these environmental factors. Following Battese & Coelli (1995), Equation 2 allows an estimation of the explanatory factors for  $\lambda_n$  while Equation 3 gives a final efficiency score that is corrected for factors that could have undermined the accuracy in the estimation of the inefficiency measure  $\lambda_n$ .

$$\lambda_n = Z_n \eta + w_n$$

$$TE_n = \exp(-\lambda_n) = \exp(-Z_n \eta - w_n)$$

where  $w_n$  is an error term that is normally distributed and is truncated at the point  $Z_n \eta$ , with mean 0 and variance  $\sigma_w^2$ .  $Z_n$  is the matrix of explanatory variables that include some explanatory factors of the inefficiency parameter  $\lambda_n$ ,  $\eta$  is a vector of parameters to be estimated and  $TE_n$  is the technical efficiency.

Equations 1 and 2 are estimated simultaneously with the maximum likelihood method, using the likelihood function suggested by Battese & Coelli (1995) and the efficiency score is calculated using Equation 3. Two main functional

forms are used in the literature for  $f(\cdot)$ : the ‘translog’ function and the Cobb Douglas function. The translog function is more flexible as it allows the frontier to be quasi-concave.  $f(\cdot)$  is then approximated by a translog function. The translog function is suitable for capturing the concave relationship between public expenditure and educational outcomes. It is assumed that an additional unit of expenditure has a lower impact compared to the previous unit.”

## Appendix B

Table 19: Results of the stochastic frontier estimations with multiple observations allowed per country, weighted by the number of observations per country.

	Model 1a	Model 1b	Model 2a	Model 2b
<b>Frontier</b>				
ln_expPC_p_avg	1.200*** (0.142)	1.150*** (0.157)	0.996*** (0.000)	0.996*** (0.000)
sq_ln_expPC_p_avg	-0.064*** (0.011)	-0.064*** (0.011)	-0.055*** (0.000)	-0.055*** (0.000)
world_region_eap			0.557*** (0.000)	0.557*** (0.000)
world_region_eca			0.389*** (0.000)	0.389*** (0.000)
world_region_lac			0.097*** (0.000)	0.097*** (0.000)
world_region_mena			0.177*** (0.000)	0.177*** (0.000)
world_region_na			0.242*** (0.000)	0.242*** (0.000)
world_region_sa			0.515*** (0.000)	0.515*** (0.000)
world_region_ssa			-0.000 (.)	-0.000 (.)
Constant	-0.864 (0.459)	-0.486 (0.561)	-0.203*** (0.000)	-0.203*** (0.000)
<b>Usigma</b>				
odashare_p_avg		17.313*** (3.348)		19.700*** (3.276)
Constant	-0.631*** (0.141)	-1.173*** (0.142)	-0.895*** (0.104)	-1.640*** (0.120)
<b>Vsigma</b>				
Constant	-4.333*** (0.580)	-5.009*** (0.557)	-36.184 (244.527)	-40.317 (566.794)
Observations	185	185	185	185
Wald chi2 (2) =	200.56	118.94	1.28e+11	1.02e+12
Prob > chi2 =	0.0000	0.0000	0.0000	0.0000
Average efficiency score:	0.608	0.616	0.667	0.667

\*Significant at 5%; \*\*significant at 1%; \*\*\*significant at 0.1%

The input and output variables are logged, and the marginal effects capture elasticities

The spend term and squared spend term are now significant at the 0.1% level across all specifications. The directions and coefficients of the spend term and squared spend term are again consistent across specifications both in this sample and with the previous unweighted sample.

Table 20: Weighted bootstrap estimates

	LICs	LMICs	UMICs	HICs
Model 1a	3,484***	3,630***	12,186	38,140
Model 1b	4,006*	4,428*	16,726	68,704
Model 2a	4,522**	4,885***	17,571***	63,666
Model 2b	4,446**	5,330***	24,671***	276,957

\*Significant at 5%; \*\*significant at 1%; \*\*\*significant at 0.1%

Note in model 2a and 2b, one or more parameters could not be estimated in 5 and 4 bootstrap replicates respectively. This occurs on the occasions where the bootstrapped sample happens to not include any of the three countries in the North America region. This means the parameters cannot be estimated for that sample but these few occurrences do not affect estimated values. Standard-error estimates include only complete replications.

Table 21: Results of the stochastic frontier estimations without the squared spend term, latest data only

	Model 1a	Model 1b	Model 2a	Model 2b
<b>Frontier</b>				
ln_expPC_p_avg	0.399*** (0.047)	0.267*** (0.052)	0.330*** (0.040)	0.375*** (0.061)
world_region_eap			0.758*** (0.150)	1.031*** (0.217)
world_region_eca			0.460** (0.161)	0.845*** (0.229)
world_region_lac			0.212 (0.145)	0.519** (0.179)
world_region_mena			0.188 (0.165)	0.525* (0.222)
world_region_na			0.214 (0.319)	0.654 (0.408)
world_region_sa			0.728*** (0.172)	0.758*** (0.230)
world_region_ssa			-0.000 (.)	0.000 (.)
Constant	1.359*** (0.389)	2.366*** (0.442)	1.510*** (0.257)	0.448 (0.362)
<b>Usigma</b>				
odashare_p_avg		17.928*** (4.015)		-1.96e+05 (2.94e+06)
Constant	-0.606* (0.268)	-1.347*** (0.285)	-0.727*** (0.214)	-4.360* (1.802)
<b>Vsigma</b>				
Constant	-2.580*** (0.419)	-3.427*** (0.542)	-3.623*** (0.576)	-1.551*** (0.145)
Observations	95	95	95	95
Wald chi2 (2) =	72.85	26.26	186.42	220.25
Prob > chi2 =	0.0000	0.0000	0.0000	0.0000
Average efficiency score:	0.623	0.636	0.649	0.944

\*Significant at 5%; \*\*significant at 1%; \*\*\*significant at 0.1%

The input and output variables are logged, and the marginal effects capture elasticities

The spend term is significant at the 0.1% level across all specifications. The directions and coefficients of the spend term and squared spend term are again consistent across specifications both in this sample and with the previous unweighted sample.

Table 22: Bootstrap estimates without squared spend term

	LICs	LMICs	UMICs	HICs
Model 1a	4,814**	4,837***	9,215	20,674
Model 1b	7,181	7,214	13,745***	30,838***
Model 2a	5,817*	5,844***	11,134***	24,980***
Model 2b	5,122	5,146	9,803	21,995

\*Significant at 5%; \*\*significant at 1%; \*\*\*significant at 0.1%

Note in model 2a and 2b, one or more parameters could not be estimated in 48 and 63 bootstrap replicates respectively. This occurs on the occasions where the bootstrapped sample happens to not include any of the three countries in the North America region. This means the parameters cannot be estimated for that sample but these few occurrences do not affect estimated values. Standard-error estimates include only complete replications.

# ENDNOTES

1. World Bank. 2019. Ending Learning Poverty : What Will It Take?. World Bank, Washington, DC. © World Bank. <https://openknowledge.worldbank.org/handle/10986/32553> License: CC BY 3.0 IGO.
2. Azevedo, Joao Pedro Wagner De; Goldemberg, Diana; Montoya, Silvia; Nayar, Reema; Rogers, F. Halsey; Saavedra, Jaime; Stacy, Brian William. 2021. Will Every Child Be Able to Read by 2030 ? Defining Learning Poverty and Mapping the Dimensions of the Challenge (English). Policy Research working paper; no. WPS 9588; COVID-19 (Coronavirus) Washington, D.C. : World Bank Group.
3. Azevedo, Joao Pedro. 2020. Learning Poverty : Measures and Simulations. Policy Research Working Paper; No. 9446. World Bank, Washington, DC. © World Bank. <https://openknowledge.worldbank.org/handle/10986/34654> License: CC BY 3.0 IGO.
4. Saavedra Chanduvi, Jaime; Aedo Inostroza, Mario Cristian; Arias Diaz, Omar S.. 2020. Realizing the Future of Learning : From Learning Poverty to Learning for Everyone, Everywhere (English). Washington, D.C. : World Bank Group.
5. Al-Samarrai, Samer; Cerdan-Infantes, Pedro; Lehe, Jonathan. 2019. Mobilizing Resources for Education and Improving Spending Effectiveness : Establishing Realistic Benchmarks Based on Past Trends. Policy Research Working Paper; No. 8773. World Bank, Washington, DC. © World Bank. <https://openknowledge.worldbank.org/handle/10986/31399> License: CC BY 3.0 IGO.
6. Miningou, Elise Wendlassida. 2019. Quality Education and the Efficiency of Public Expenditure : A Cross-Country Comparative Analysis. Policy Research Working Paper; No. 9077. World Bank, Washington, DC. © World Bank. <https://openknowledge.worldbank.org/handle/10986/33021> License: CC BY 3.0 IGO.
7. The only country with relevant learning poverty data that this is not available for is Yemen. As this is only one observation, we do not try to resolve this and instead Yemen is not included in the later model.
8. The only country with relevant learning poverty data that this is not available for is Japan. As this is only one observation, we do not try to resolve this and instead Japan is not included in the later model.
9. Azevedo, Joao Pedro. 2020. Learning Poverty : Measures and Simulations. Policy Research Working Paper; No. 9446. World Bank, Washington, DC. © World Bank.
10. Global Partnership for Education, 2020. Methodology for the Replenishment Indicators. (Last revision: November 2020).
11. Global Partnership for Education, 2020. Methodology for the Replenishment Indicators. (Last revision: November 2020).
12. Global Partnership for Education, 2020. Methodology for the Replenishment Indicators. (Last revision: November 2020).
13. Miningou, Elise Wendlassida. 2019. Quality Education and the Efficiency of Public Expenditure : A Cross-Country Comparative Analysis. Policy Research Working Paper; No. 9077. World Bank, Washington, DC. © World Bank. <https://openknowledge.worldbank.org/handle/10986/33021> License: CC BY 3.0 IGO.
14. Hanushek and L. Woessman, 2007. The Role of Education Quality in Economic Growth. Policy Research Working Paper 4122, World Bank, Washington, DC.
15. M. Schäferhoff et al., "Estimating the Economic Returns of Education From a Health Perspective," Prepared by SEEK Development for the International Commission on Financing Global Education Opportunity as a background paper for The Learning Generation, Berlin, Germany, 2016, <http://report.educationcommission.org/wp-content/uploads/2016/11/Estimatingthe-Economic>Returns-of-Education-from-a-Health-Perspective.pdf>.
16. Q. Wodon et al., Educating Girls and Ending Child Marriage: A Priority for Africa, The Cost of Not Educating Girls Note Series (Washington, DC: World Bank, 2018), <http://documents1.worldbank.org/curated/en/268251542653259451/pdf/132200-WPP168381-PUBLIC-11-20-18-Africa-GE-CM-Conference-Edition2.pdf>.
17. Alderman and Headey, 2017. How Important is Parental Education for Child Nutrition? The International Food Policy Research Institute, USA